Int J Theor Phys (2009) 48: 568–578
DOI 10.1007/s10773-008-9832-9

Cylindrically Symmetric Inhomogeneous Universes with a Cloud of Strings

Anil Kumar Yadav · Vineet K. Yadav · Lallan Yadav

Received: 9 June 2008 / Accepted: 8 August 2008 / Published online: 3 September 2008

Abstract Cylindrically symmetric inhomogeneous string cosmological models are investigated in presence of string fluid as a source of matter. To get the three types of exact solutions of Einstein's field equations we assume $A = f(x)k(t)$, $B = g(x)\ell(t)$ and $C = h(x)\ell(t)$. Some physical and geometric aspects of the models are discussed.

Keywords String · Inhomogeneous universe · Cylindrical symmetry

1 Introduction

In recent years, there has been considerable interest in string cosmology because cosmic strings play an important role in the study of the early universe. These strings arise during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories [14–16, 36, 47, 48]. Moreover, the investigation of cosmic strings and their physical processes near such strings has received wide attention because it is believed that cosmic strings give rise to density perturbations which lead to formation of galaxies [36, 46]. These cosmic strings have stress energy and couple to the gravitational field. Therefore, it is interesting to study the gravitational effect which arises from strings by using Einstein's equations.

The general treatment of strings was initiated by Letelier [20, 21] and Stachel [33]. Letelier [20] obtained the general solution of Einstein's field equations for a cloud of strings

A.K. Yadav (✉)
Department of Physics, Anand Engineering College, Keetham, Agra 282 007, India
e-mail: abanilyadav@yahoo.co.in

V.K. Yadav · L. Yadav
Department of Physics, D.D.U. Gorakhpur University, 273 009, Gorakhpur, India

V.K. Yadav
e-mail: vineetky@yahoo.co.in

L. Yadav
e-mail: lallan_yadav07@yahoo.co.in

with spherical, plane and a particular case of cylindrical symmetry. Letelier [21] also obtained massive string cosmological models in Bianchi type-I and Kantowski-Sachs space-times. Banerjee et al. [8] have investigated an axially symmetric Bianchi type I string dust cosmological model in presence and absence of magnetic field. Exact solutions of string cosmology for Bianchi type-II, -VI$_0$, -VIII and -IX space-times have been studied by Krori et al. [18] and Wang [37]. Wang [38–41] and Yadav et al. [42, 43] have investigated bulk viscous string cosmological models in different space-times. Bali et al. [1–5, 7] have obtained Bianchi type-I, -III, -V and type-IX string cosmological models in general relativity. The string cosmological models with a magnetic field are discussed by Chakraborty [13], Tikekar and Patel [34, 35], Patel and Maharaj [22]. Ram and Singh [29] obtained some new exact solution of string cosmology with and without a source free magnetic field for Bianchi type I space-time in the different basic form considered by Carminati and McIntosh [12]. Singh and Singh [30] investigated string cosmological models with magnetic field in the context of space-time with G_3 symmetry. Singh [31, 32] has studied string cosmology with electromagnetic fields in Bianchi type-II, -VIII and -IX space-times. Lidsey, Wands and Copeland [19] have reviewed aspects of super string cosmology with the emphasis on the cosmological implications of duality symmetries in the theory. Yavuz et al. [44] have examined charged strange quark matter attached to the string cloud in the spherical symmetric space-time admitting one-parameter group of conformal motion. Kaluza-Klein cosmological solutions are obtained by Yilmaz [45] for quark matter attached to the string cloud in the context of general relativity. Recently Pradhan et al. [25–27] have studied higher dimensional strange quark matter coupled to the string cloud with electromagnetic field admitting one parameter group of conformal motion.

Cylindrically symmetric space-time play an important role in the study of the universe on a scale in which anisotropy and inhomogeneity are not ignored. Inhomogeneous cylindrically symmetric cosmological models have significant contribution in understanding some essential features of the universe such as the formation of galaxies during the early stages of their evolution. Bali and Tyagi [6] and Pradhan et al. [23, 24] have investigated cylindrically symmetric inhomogeneous cosmological models in presence of electromagnetic field. Barrow and Kunze [9, 10] found a wide class of exact cylindrically symmetric flat and open inhomogeneous string universes. In their solutions all physical quantities depend on at most one space coordinate and the time. The case of cylindrical symmetry is natural because of the mathematical simplicity of the field equations whenever there exists a direction in which the pressure equal to energy density.

Recently Baysal et al. [11], Kilinc and Yavuz [17], Pradhan [28] and Pradhan et al. [26, 27] have investigated some string cosmological models in cylindrically symmetric inhomogeneous universe. In this paper, we have revisited their solutions and obtained a new class of solutions. Here, we extend our understanding of inhomogeneous string cosmologies by investigating the simple models of non-linear cylindrically symmetric inhomogeneities outlined above. This paper is organized as follows: The metric and field equations are presented in Sect 2. In Sect. 3, we deal with the solution of the field equations in three different cases. Finally, the results are discussed in Sect. 4. The solutions obtained in this paper are new and different from the other author's solutions.

2 The Metric and Field Equations

We consider the metric in the form

$$ds^2 = A^2(dx^2 - dt^2) + B^2 dy^2 + C^2 dz^2, \tag{1}$$

where A, B and C are functions of x and t. The Einstein's field equations for a cloud of strings read as [21]

$$G_i^j \equiv R_i^j - \frac{1}{2}Rg_i^j = -(\rho u_i u^j - \lambda x_i x^j), \tag{2}$$

where u_i and x_i satisfy conditions

$$u^i u_i = -x^i x_i = -1, \tag{3}$$

and

$$u^i x_i = 0. \tag{4}$$

Here, ρ is the rest energy of the cloud of strings with massive particles attached to them. $\rho = \rho_p + \lambda$, ρ_p being the rest energy density of particles attached to the strings and λ the density of tension that characterizes the strings. The unit space-like vector x^i represents the string direction in the cloud, i.e. the direction of anisotropy and the unit time-like vector u^i describes the four-velocity vector of the matter satisfying the following conditions

$$g_{ij} u^i u^j = -1. \tag{5}$$

In the present scenario, the commoving coordinates are taken as

$$u^i = \left(0, 0, 0, \frac{1}{A}\right) \tag{6}$$

and choose x^i parallel to x-axis so that

$$x^i = \left(\frac{1}{A}, 0, 0, 0\right). \tag{7}$$

The Einstein's field equations (2) for the line-element (1) lead to the following system of equations:

$$G_1^1 \equiv \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{A_1}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) - \frac{B_1C_1}{BC} + \frac{B_4C_4}{BC} = \lambda A^2, \tag{8}$$

$$G_2^2 \equiv \left(\frac{A_4}{A}\right)_4 - \left(\frac{A_1}{A}\right)_1 + \frac{C_{44}}{C} - \frac{C_{11}}{C} = 0, \tag{9}$$

$$G_3^3 \equiv \left(\frac{A_4}{A}\right)_4 - \left(\frac{A_1}{A}\right)_1 + \frac{B_{44}}{B} - \frac{B_{11}}{B} = 0, \tag{10}$$

$$G_4^4 \equiv -\frac{B_{11}}{B} - \frac{C_{11}}{C} + \frac{A_1}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) + \frac{A_4}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) - \frac{B_1C_1}{BC} + \frac{B_4C_4}{BC} = \rho A^2, \tag{11}$$

$$G_4^1 \equiv \frac{B_{14}}{B} + \frac{C_{14}}{C} - \frac{A_4}{A}\left(\frac{B_1}{B} + \frac{C_1}{C}\right) - \frac{A_1}{A}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) = 0, \tag{12}$$

where the sub indices 1 and 4 in A, B, C and elsewhere denote differentiation with respect to x and t, respectively.

The velocity field u^i is irrotational. The scalar expansion θ, shear scalar σ^2, acceleration vector $\dot{u}_i$ and proper volume V^3 are respectively found to have the following expressions:

$$\theta = u^i_{;i} = \frac{1}{A}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right), \tag{13}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\theta^2 - \frac{1}{A^2}\left(\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{C_4 A_4}{CA}\right), \tag{14}$$

$$\dot{u}_i = u_{i;j}u^j = \left(\frac{A_1}{A}, 0, 0, 0\right), \tag{15}$$

$$V^3 = \sqrt{-g} = A^2 BC, \tag{16}$$

where g is the determinant of the metric (1). Using the field equations and the relations (13) and (14) one obtains the Raychaudhuri's equation as

$$\dot{\theta} = \dot{u}^i_{;i} - \frac{1}{3}\theta^2 - 2\sigma^2 - \frac{1}{2}\rho_p, \tag{17}$$

where dot denotes differentiation with respect to t and

$$R_{ij}u^i u^j = \frac{1}{2}\rho_p. \tag{18}$$

With the help of (1)–(7), the Bianchi identity $(T^{ij}_{;j})$ reduced to two equations:

$$\rho_4 - \frac{A_4}{A}\lambda + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)\rho = 0 \tag{19}$$

and

$$\lambda_1 - \frac{A_1}{A}\rho + \left(\frac{A_1}{A} + \frac{B_1}{B} + \frac{C_1}{C}\right)\lambda = 0. \tag{20}$$

Thus due to all the three (strong, weak and dominant) energy conditions, one finds $\rho \geq 0$ and $\rho_p \geq 0$, together with the fact that the sign of λ is unrestricted, it may take values positive, negative or zero as well.

3 Solutions of the Field Equations

From (9) and (10), we obtain

$$\frac{B_{44}}{B} - \frac{B_{11}}{B} = \frac{C_{44}}{C} - \frac{C_{11}}{C}, \tag{21}$$

and

$$2\left(\frac{A_4}{A}\right)_4 - 2\left(\frac{A_1}{A}\right)_1 + \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{B_{11}}{B} - \frac{C_{11}}{C} = 0. \tag{22}$$

To get determinate solution we assume

$$\begin{aligned} A &= f(x)k(t), \\ B &= g(x)\ell(t), \\ C &= h(x)\ell(t) \end{aligned} \tag{23}$$

and

$$\frac{f_1}{f} = m \text{ (constant)}. \tag{24}$$

Using (23) in (21), we get

$$\frac{g_{11}}{g} = \frac{h_{11}}{h}. \tag{25}$$

Equations (22), (23) and (25), we have

$$\left(\frac{k_{44}}{k} - \frac{k_4^2}{k^2} + \frac{\ell_{44}}{\ell}\right) = \left(\frac{f_{11}}{f} - \frac{f_1^2}{f^2} + \frac{g_{11}}{g}\right) = s \text{ (constant)}. \tag{26}$$

Using (24) in right hand side of (26) leads to

$$\frac{g_{11}}{g} = s, \tag{27}$$

which with the use of (25) leads to

$$\frac{g_{11}}{g} = \frac{h_{11}}{h} = s. \tag{28}$$

Equation (28) leads to

$$g = h = \begin{cases} c_1 \cosh(\sqrt{s}x + x_0) & \text{when } s > 0, \\ (c_1 x + c_2) & \text{when } s = 0, \\ c_1 \cos(\sqrt{|s|}x + x_0) & \text{when } s < 0, \end{cases} \tag{29}$$

where c_1, c_2 and x_0 are constants of integration.
Using (23) in (12), we have

$$\frac{\frac{\ell_4}{\ell}}{\frac{k_4}{k}} = \frac{\frac{g_1}{g} + \frac{h_1}{h}}{\frac{g_1}{g} + \frac{h_1}{h} - \frac{2f_1}{f}} = b \text{ (constant)}. \tag{30}$$

Equation (30) leads to

$$\frac{\ell_4}{\ell} = b\frac{k_4}{k}, \tag{31}$$

which after integration gives

$$\ell = nk^b, \tag{32}$$

where n is constant of integration. Using (32) in (26), we have

$$\frac{\ell_{44}}{\ell} - \frac{1}{(1+b)}\frac{\ell_4^2}{\ell^2} = N, \tag{33}$$

where $N = \frac{sb}{(1+b)}$. Equation (33) leads to

$$\ell = \begin{cases} c_3^{\frac{(1+b)}{b}} \cosh^{\frac{(1+b)}{b}}(\sqrt{r}t + t_0) & \text{when } r > 0, \\ (c_3 t + c_4)^{\frac{(1+b)}{b}} & \text{when } r = 0, \\ c_3^{\frac{(1+b)}{b}} \cos^{\frac{(1+b)}{b}}(\sqrt{|r|}t + t_0) & \text{when } r < 0, \end{cases} \tag{34}$$

where $r = \frac{Nb}{(1+b)}$ and c_3, c_4, t_0 are constants of integration.

Hence from (32) we obtain

$$k = \begin{cases} \frac{c_3^{\frac{(1+b)}{b^2}}}{n^{\frac{1}{b}}} \cosh^{\frac{(1+b)}{b^2}}(\sqrt{r}t + t_0) & \text{when } r > 0, \\ \frac{(c_3 t + c_4)^{\frac{(1+b)}{b^2}}}{n^{\frac{1}{b}}} & \text{when } r = 0, \\ \frac{c_3^{\frac{(1+b)}{b^2}}}{n^{\frac{1}{b}}} \cos^{\frac{(1+b)}{b^2}}(\sqrt{|r|}t + t_0) & \text{when } r < 0. \end{cases} \tag{35}$$

Equation (24) leads to

$$f = e^{mx}. \tag{36}$$

Now we consider the following three cases.

3.1 When $s > 0$ and $r > 0$

In this case we have

$$A = \alpha e^{mx} \cosh^{\frac{(1+b)}{b}}(\sqrt{r}t + t_0), \tag{37}$$

$$B = C = \beta \cosh(\sqrt{s}x + x_0) \cosh^{\frac{(1+b)}{b}}(\sqrt{r}t + t_0), \tag{38}$$

where $\alpha = \frac{c_3^{\frac{(1+b)}{b^2}}}{n^{\frac{1}{b}}}$ and $\beta = c_1 c_3^{\frac{(1+b)}{b}}$.

After using suitable transformation of the co-ordinates

$$X = x + \frac{x_0}{\sqrt{s}}, \qquad Y = y, \qquad Z = z, \qquad T = t + \frac{t_0}{\sqrt{r}}, \tag{39}$$

the metric (1) reduces to the form

$$ds^2 = \alpha^2 e^{2m(X - \frac{x_0}{\sqrt{s}})} \cosh^{\frac{2(1+b)}{b^2}}(\sqrt{r}T)(dX^2 - dT^2) \\ + \beta^2 \cosh^2(\sqrt{s}X) \cosh^{\frac{2(1+b)}{b}}(\sqrt{r}T)(dY^2 + dZ^2). \tag{40}$$

In this case the physical parameters, i.e. the string tension density (λ), the energy density (ρ), the particle density (ρ_p) and the kinematical parameters, i.e. the scalar of expansion (θ), shear tensor (σ), the acceleration vector ($\dot{u}_i$) and the proper volume (V^3) for the model (40) are given by

$$\lambda = \frac{1}{\alpha^2 e^{2m(X - \frac{x_0}{\sqrt{s}})} \cosh^{\frac{2(1+b)}{b^2}}(\sqrt{r}T)} \left[\frac{2(1+b)r}{b^2} + \frac{(1+b)(b^2+b-2)r}{b^3} \tanh^2(\sqrt{r}T) \right.$$

$$- \sqrt{s}\tanh(\sqrt{s}X)\{2me^{m(X-\frac{x_0}{\sqrt{s}})} + \sqrt{s}\tanh(\sqrt{s}X)\}\Bigg], \tag{41}$$

$$\rho = \frac{1}{\alpha^2 e^{2m(X-\frac{x_0}{\sqrt{s}})}\cosh^{\frac{2(1+b)}{b^2}}(\sqrt{r}T)}\Bigg[\frac{(1+b)^2(2+b)r}{b^3}\tanh^2(\sqrt{r}T)$$

$$+ \sqrt{s}\tanh(\sqrt{s}X)\{2me^{m(X-\frac{x_0}{\sqrt{s}})} - \sqrt{s}\tanh(\sqrt{s}X)\} - 2s\Bigg], \tag{42}$$

$$\rho_p = \frac{1}{\alpha^2 e^{2m(X-\frac{x_0}{\sqrt{s}})}\cosh^{\frac{2(1+b)}{b^2}}(\sqrt{r}T)}\Bigg[\frac{2(1+b)(2+b)r}{b^3}\tanh^2(\sqrt{r}T)$$

$$+ 4\sqrt{s}e^{m(X-\frac{x_0}{\sqrt{s}})}\tanh(\sqrt{s}X) - \frac{2(1+b)r}{b^2} - 2s\Bigg], \tag{43}$$

$$\theta = \frac{(1+b)(1+2b)\sqrt{r}\tanh(\sqrt{r}T)}{b^2\alpha e^{m(X-\frac{x_0}{\sqrt{s}})}\cosh^{\frac{(1+b)}{b^2}}(\sqrt{r}T)}, \tag{44}$$

$$\sigma^2 = \frac{(1+b)^2(b-1)^2 r\tanh^2(\sqrt{r}T)}{3b^4\alpha^2 e^{2m(X-\frac{x_0}{\sqrt{s}})}\cosh^{\frac{2(1+b)}{b^2}}(\sqrt{r}T)}, \tag{45}$$

$$\dot{u}_i = (m, 0, 0, 0), \tag{46}$$

$$V^3 = \sqrt{-g} = \alpha^2\beta^2 e^{2m(X-\frac{x_0}{\sqrt{s}})}\cosh^2(\sqrt{s}X)\cosh^{\frac{2(1+b)}{b^2}}(\sqrt{r}T). \tag{47}$$

From (44) and (45), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{(b-1)^2}{3(2b+1)^2} = \text{constant}. \tag{48}$$

The energy conditions $\rho \geq 0$ and $\rho_p \geq 0$ are satisfied for the model (40). The conditions $\rho \geq 0$ and $\rho_p \geq 0$ lead to

$$\frac{(1+b)^2(2+b)r}{b^3}\tanh^2(\sqrt{r}T) + \sqrt{s}\tanh(\sqrt{s}X)$$

$$\times \left\{2me^{m(X-\frac{x_0}{\sqrt{s}})} - \sqrt{s}\tanh(\sqrt{s}X)\right\} \geq 2s \tag{49}$$

and

$$\frac{2(1+b)(2+b)r}{b^3}\tanh^2(\sqrt{r}T) + 4\sqrt{s}e^{m(X-\frac{x_0}{\sqrt{s}})}\tanh(\sqrt{s}X) \geq \frac{2(1+b)r}{b^2} + 2s \tag{50}$$

respectively. From (41), we observe that the string tension density $\lambda \geq 0$ leads to

$$\tanh^2(\sqrt{r}T) \geq \frac{b^3\sqrt{s}\tanh(\sqrt{s}X)}{(1+b)(b^2+b-2)}\left\{2me^{m(X-\frac{x_0}{\sqrt{s}})} + \sqrt{s}\tanh(\sqrt{s}X)\right\} - \frac{2br}{(b^2+b-2)}. \tag{51}$$

The model (40) are shearing, accelerating and non-rotating. For $b < -1$, when $T \to 0$, $\theta \to 0$ and when $T \to \infty$, $\theta \to \infty$. Hence for $b < -1$, the model is expanding. Since $\frac{\sigma}{\theta} =$ constant, therefore the model does not approach isotropy. But we observe that for $b = 1$, shear scalar is zero and hence the model isotropizes for this value of b. We also observe that when $T \to \infty$, the proper volume $V^3 \to \infty$ and $\rho \to 0$. Hence volume increases when T increases and the energy density is a decreasing function of T.

3.2 When $s = 0$ and $r = 0$

In this case we obtain

$$A = \frac{(c_3 t + c_4)^{\frac{(1+b)}{b^2}}}{n^{\frac{1}{b}}} e^{mx}, \tag{52}$$

$$B = C = (c_1 x + c_2)(c_3 t + c_4)^{\frac{(1+b)}{b}}. \tag{53}$$

By using suitable transformation, the metric (1) reduces to the form

$$ds^2 = \alpha^2 e^{2m(X-\delta)} T^{\frac{2(1+b)}{b^2}} (dX^2 - dT^2) + \beta^2 X^2 T^{\frac{2(1+b)}{b}} (dY^2 + dZ^2), \tag{54}$$

where $x = X - \delta$, $y = Y$, $z = Z$, $t + \frac{c_4}{c_3} = T$, $\delta = \frac{c_2}{c_1}$. α and β are already defined in previous section.

In this case the physical parameters, i.e. the string tension density (λ), the energy density (ρ), the particle density (ρ_p) and the kinematical parameters, i.e. the scalar of expansion (θ), shear tensor (σ), the acceleration vector ($\dot{u}_i$) and the proper volume (V^3) for the model (40) are given by

$$\lambda = \frac{1}{\alpha^2 e^{2m(X-\delta)} T^{\frac{2(1+b)}{b^2}}} \left[\frac{(1+b)\{b(3+b) - 2(1+b)\}}{b^3} \frac{1}{T^2} - \frac{2m}{X} - \frac{1}{X^2} \right], \tag{55}$$

$$\rho = \frac{1}{\alpha^2 e^{2m(X-\delta)} T^{\frac{2(1+b)}{b^2}}} \left[\frac{(1+b)^2(2+b)}{b^3} \frac{1}{T^2} + \frac{2m}{X} - \frac{1}{X^2} \right], \tag{56}$$

$$\rho_p = \frac{1}{\alpha^2 e^{2m(X-\delta)} T^{\frac{2(1+b)}{b^2}}} \left[\frac{(1+b)(4-b)}{b^3} \frac{1}{T^2} + \frac{4m}{X} \right], \tag{57}$$

$$\theta = \frac{(1+b)(1+2b)}{b^2} \frac{1}{\alpha e^{m(X-\delta)} T^{\frac{b(1+b)+1}{b}}}, \tag{58}$$

$$\sigma^2 = \frac{(1+b)^2(b-1)^2}{b^3} \frac{1}{\alpha^2 e^{2m(X-\delta)} T^{\frac{2b(1+b)+2}{b^2}}}, \tag{59}$$

$$\dot{u}_i = (m, 0, 0, 0), \tag{60}$$

$$V^3 = \sqrt{-g} = \alpha^2 \beta^2 e^{2m(X-\delta)} X^2 T^{\frac{2(1+b)^2}{b^2}}, \tag{61}$$

$$\frac{\sigma^2}{\theta^2} = \frac{b(b-1)^2}{(1+2b)^2} = \text{constant}. \tag{62}$$

The energy conditions $\rho \geq 0$ and $\rho_p \geq 0$ are satisfied for the model (54). The conditions $\rho \geq 0$ and $\rho_p \geq 0$ lead to

$$\frac{(1+b)^2(2+b)}{b^3} \frac{1}{T^2} \geq \frac{1}{X^2} - \frac{2m}{X^2}, \tag{63}$$

and

$$\frac{(1+b)(4-b)}{b^3}\frac{1}{T^2}+\frac{4m}{X}\geq 0, \tag{64}$$

respectively. From (55), we observe that the string tension density $\lambda \geq 0$ leads to

$$\frac{(1+b)\{b(3+b)-2(1+b)\}}{b^3}\frac{1}{T^2}\geq \frac{2m}{X}+\frac{1}{X^2}. \tag{65}$$

The model (54) has singularity at $T=0$. The model starts with a big bang singularity at $T=0$ and continue to expand till $T=\infty$. Since $\frac{\sigma}{\theta}=$ constant, therefore the model does not approach isotropy. But we observe that for $b=1$, shear scalar is zero and hence the model isotropizes for this value of b. We also observe that when $T\to\infty$, the proper volume $V^3\to\infty$ and $\rho\to 0$. Hence volume increases when T increases and the energy density is a decreasing function of T. Generally the model (54) represents an expanding, shearing, accelerating and non-rotating universe.

3.3 When $s<0$ and $r<0$

In this case we obtain

$$A=\alpha e^{mx}\cos^{\frac{(b+1)}{b^2}}(\sqrt{|r|}t+t_0), \tag{66}$$

$$B=C=\beta\cos(\sqrt{|s|}x+x_0)\cos^{\frac{(b+1)}{b}}(\sqrt{|r|})t+t_0). \tag{67}$$

By using suitable transformation, the metric (1) reduces to the form

$$ds^2=\alpha^2 e^{2m(X-\frac{x_0}{\sqrt{s}})}\cos^{\frac{2(b+1)}{b^2}}(\sqrt{|r|}T)(dX^2-dT^2) + \beta^2\cos^2(\sqrt{|s|}X)\cos^{\frac{2(b+1)}{b^2}}(\sqrt{|r|}T)(dY^2+dZ^2). \tag{68}$$

In this case the physical parameters, i.e. the string tension density (λ), the energy density (ρ), the particle density (ρ_p) and the kinematical parameters, i.e. the scalar of expansion (θ), shear tensor (σ), the acceleration vector ($\dot{u}_i$) and the proper volume (V^3) for the model (40) are given by

$$\lambda=\frac{1}{Ł^2 e^{2m(X-\frac{x_0}{\sqrt{|s|}})}\cos^{\frac{2(b+1)}{b^2}}(\sqrt{|r|}T)} \times\left[-\frac{2(b+1)|r|}{b}+\frac{(b+1)(b^2+b-2)}{b^3}|r|\tan^2(\sqrt{|r|}T) + \sqrt{|s|}\tan(\sqrt{|s|}X)\left\{2ne^{m(X-\frac{x_0}{\sqrt{|s|}})}-\sqrt{|s|}\tan(\sqrt{|s|}X)\right\}\right], \tag{69}$$

where $\alpha=-Ł$, $Ł>0$.

$$\rho=\frac{1}{Ł^2 e^{2m(X-\frac{x_0}{\sqrt{|s|}})}\cos^{\frac{2(b+1)}{b^2}}(\sqrt{|r|}T)} \times\left[\frac{(b+2)(b+1)^2}{b^3}|r|\tan^2(\sqrt{|r|}T)+2|s|\right.$$

$$- \sqrt{|s|}\tan(\sqrt{|s|}X)\left\{2ne^{m(X-\frac{x_0}{\sqrt{|s|}})} + \sqrt{|s|}\tan(\sqrt{|s|}X)\right\}\Bigg], \tag{70}$$

$$\rho_p = \frac{1}{Ł^2 e^{2m(X-\frac{x_0}{\sqrt{|s|}})}\cos^{\frac{2(b+1)}{b^2}}(\sqrt{|r|}T)} \times \left[\frac{2(b+2)(b+1)}{b^3}|r|\tan^2(\sqrt{|r|}T) + 2|s| - 4\sqrt{|s|}me^{m(X-\frac{x_0}{\sqrt{|s|}})}\tan(\sqrt{|s|}X) + \frac{2(b+1)|r|}{b}\right], \tag{71}$$

$$\theta = \frac{(b+1)(2b+1)|r|\tan(\sqrt{|r|}T)}{b^2 Ł e^{m(X-\frac{x_0}{\sqrt{|s|}})}\cos^{\frac{(b+1)}{b^2}}(\sqrt{|r|}T),} \tag{72}$$

$$\sigma^2 = \frac{(b-1)^2(b+1)^2|r|\tan^2(\sqrt{|r|}T)}{3b^4 Ł^2 e^{2m(X-\frac{x_0}{\sqrt{|s|}})}\cos^{\frac{2(b+1)}{b^2}}(\sqrt{|r|}T)}, \tag{73}$$

$$\dot{u}_i = (m, 0, 0, 0), \tag{74}$$

$$V^3 = Ł^2\beta^2 e^{2m(X-\frac{x_0}{\sqrt{|s|}})}\cos^2(\sqrt{|s|}X)\cos^{\frac{2(b+1)}{b^2}}(\sqrt{|r|}T). \tag{75}$$

From (72) and (73), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{(b-1)^2}{3(2b+1)^2} = \text{constant}. \tag{76}$$

The model (68) starts expanding at $T = 0$ and attains its maximum value at $T = \frac{\pi}{4\sqrt{|r|}}$. After that θ decreases to attain its minimum value at $T = \frac{3\pi}{4\sqrt{|r|}}$. The model oscillates with period $\frac{\pi}{2\sqrt{|r|}}$. The model is shearing and non-rotating. Since $\frac{\sigma}{\theta} =$ constant, therefore the model does not approach isotropy. But we observe that for $b = 1$, shear scalar vanishes. Hence $b = 1$ is the isotropy condition.

4 Concluding Remarks

In this paper we have obtained a new class of exact solutions of Einstein's field equations in cylindrically symmetric inhomogeneous space-time with string fluid. The models (40) and (54), in general, are expanding, shearing and non-rotating. The model (68) is oscillating, shearing and non-rotating. All these three models do not isotropize. The model (54) is singular whereas the model (40) is non-singular. It is important to note here that the models (40), (54) and (68) reduce to homogeneous universe when $m = 0$ and $s = 0$. This shows that for $m = 0$ and $s = 0$, inhomogeneity dies out. In this case all these models are non-accelerating.

Acknowledgements The authors (AKY and VKY) thank the Harish-Chandra Research Institute, Allahabad, India for providing facility where this work was carried out. Authors would like to thank to A. Pradhan for his fruitful discussions.

References

1. Bali, R., Anjali: Astrophys. Space Sci. **302**, 201 (2006)
2. Bali, R., Dave, S.: Astrophys. Space Sci. **288**, 503 (2001)
3. Bali, R., Singh, D.K.: Astrophys. Space Sci. **300**, 387 (2005)
4. Bali, R., Pradhan, A.: Chin. Phys. Lett. **24**, 585 (2007)
5. Bali, R., Pareek, U.K., Pradhan, A.: Chin. Phys. Lett. **24**, 2455 (2007)
6. Bali, R., Tyagi, A.: Gen. Relativ. Gravit. **21**, 797 (1989)
7. Bali, R., Upadhaya, R.D.: Astrophys. Space Sci. **283**, 97 (2003)
8. Banerjee, A., Sanyal, A.K., Chakraborty, S.: Pramana—J. Phys. **34**, 1 (1990)
9. Barrow, J.D., Kunze, K.E.: Phys. Rev. D **56**, 741 (1997)
10. Barrow, J.D., Kunze, K.E.: Phys. Rev. D **57**, 2255 (1998)
11. Baysal, H., Yavuz, I., Tarhan, I., Camci, U., Yilmaz, I.: Turk. J. Phys. **25**, 283 (2001)
12. Carminati, J., McIntosh, C.B.G.: J. Phys. A Math. Gen. **13**, 953 (1980)
13. Chakraborty, S.: Ind. J. Pure Appl. Phys. **29**, 31 (1980)
14. Everett, A.E.: Phys. Rev. **24**, 858 (1981)
15. Kibble, T.W.B.: J. Phys. A Math. Gen. **9**, 1387 (1976)
16. Kibble, T.W.B.: Phys. Rep. **67**, 183 (1980)
17. Kilinc, C.B., Yavuz, I.: Astrophys. Space Sci. **238**, 239 (1996)
18. Krori, K.D., Chaudhury, T., Mahanta, C.R., Mazumder, A.: Gen. Relativ. Gravit. **22**, 123 (1990)
19. Lidsey, J.E., Wands, D., Copeland, E.J.: Phys. Rep. **337**, 343 (2000)
20. Letelier, P.S.: Phys. Rev. D **20**, 1249 (1979)
21. Letelier, P.S.: Phys. Rev. D **28**, 2414 (1983)
22. Patel, L.K., Maharaj, S.D.: Pramana—J. Phys. **47**, 1 (1996)
23. Pradhan, A., Chakrabarty, I., Saste, N.N.: Int. J. Mod. Phys. D **10**, 741 (2001)
24. Pradhan, A., Singh, P.K., Jotania, K.: Czech. J. Phys. **56**, 641 (2006)
25. Pradhan, A., Khadekar, G.S., Mishra, M.K., Kumbhare, S.: Chin. Phys. Lett. **24**, 3013 (2007)
26. Pradhan, A., Yadav, A.K., Singh, R.P., Singh, V.K.: Astrophys. Space Sci. **312**, 145 (2007)
27. Pradhan, A., Mishra, M.K., Yadav, A.K.: arXiv:gr-qc/0705.1765 (2007)
28. Pradhan, A.: Fizika B (Zagreb) **16**, 205 (2007)
29. Ram, S., Singh, T.K.: Gen. Relativ. Gravit. **27**, 1207 (1995)
30. Singh, G.P., Singh, T.: Gen. Relativ. Gravit. **31**, 371 (1999)
31. Singh, G.P.: Nuovo Cim. B **110**, 1463 (1995)
32. Singh, G.P.: Pramana—J. Phys. **45**, 189 (1995)
33. Stachel, J.: Phys. Rev. D **21**, 2171 (1980)
34. Tikekar, R., Patel, L.K.: Gen. Relativ. Gravit. **24**, 397 (1992/1994)
35. Tikekar, R., Patel, L.K.: Pramana—J. Phys. **42**, 483 (1994)
36. Vilenkin, A.: Phys. Rev. D **24**, 2082 (1981)
37. Wang, X.X.: Chin. Phys. Lett. **20**, 615 (2003)
38. Wang, X.X.: Astrophys. Space Sci. **293**, 933 (2004)
39. Wang, X.X.: Chin. Phys. Lett. **21**, 1205 (2004)
40. Wang, X.X.: Chin. Phys. Lett. **22**, 29 (2005)
41. Wang, X.X.: Chin. Phys. Lett. **23**, 1702 (2006)
42. Yadav, M.K., Rai, A., Pradhan, A.: Int. J. Theor. Phys. **46**, 2677 (2007)
43. Yadav, M.K., Pradhan, A., Singh, S.K.: Astrophys. Space Sci. **311**, 423 (2007)
44. Yavuz, I., Yilmaz, I., Baysal, H.: Int. J. Mod. Phys. D **14**, 1365 (2005)
45. Yilmaz, I.: Gen. Relativ. Gravit. **38**, 1397 (2006)
46. Zel'dovich, Ya.B.: Mon. Not. R. Astron. Soc. **192**, 663 (1980)
47. Zel'dovich, Ya.B., Kobzarev, I.Yu., Okun, L.B.: Zh. Eksp. Teor. Fiz. **67**, 3 (1975)
48. Zel'dovich, Ya.B., Kobzarev, I.Yu., Okun, L.B.: Sov. Phys. JETP **40**, 1 (1975)